DOUBLE ANGLES AND PROOF

Using $\sin 2A = 2\sin A \cos A$, $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$,
and $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Examples: Prove that

1) $\frac{\sin 2A}{1+\cos 2A} = \tan A$

$$
\begin{aligned}
\text{LHS} &= \frac{\sin 2A}{1+\cos 2A} \\
&= \frac{2\sin A \cos A}{1+(2\cos^2 A-1)} \\
&= \frac{2\sin A \cos A}{2\cos^2 A} \\
&= \frac{\sin A}{\cos A} \\
&= \tan A \\
&= \text{RHS}
\end{aligned}
$$

2) $\frac{1-\cos 4\theta}{1+\cos 4\theta} + 1 = \sec^2 2\theta$

$$
\begin{aligned}
\text{LHS} &= \frac{1-\cos 4\theta}{1+\cos 4\theta} + 1 = \frac{1-(1-2\sin^2 2\theta)}{1+(2\cos^2 2\theta-1)} + 1 = \frac{2\sin^2 2\theta}{2\cos^2 2\theta} + 1 = \tan^2 2\theta + 1 \\
&= \sec^2 2\theta = \text{RHS}
\end{aligned}
$$

3) $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{\cos 2\theta}{1+\sin 2\theta}$

Using $a^2 - b^2 = (a+b)(a-b)$

$$
\begin{aligned}
\text{RHS} &= \frac{\cos 2\theta}{1+\sin 2\theta} = \frac{\cos^2\theta-\sin^2\theta}{1+2\sin\theta\cos\theta} = \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta} = \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta+\sin\theta)} \\
&= \frac{(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)} = \text{LHS}
\end{aligned}
$$

Using $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

Or

$$
\begin{aligned}
\text{LHS} &= \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta+\sin\theta)} = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta} = \frac{\cos^2\theta-\sin^2\theta}{1+2\sin\theta\cos\theta} \\
&= \frac{\cos 2\theta}{1+\sin 2\theta} = \text{RHS.}
\end{aligned}
$$

Delta Ex 34.8 pg 326, Extension: Delta Ex 34.7